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ORIGINAL ARTICLE

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A MATHEMATICAL MODEL OF NANOPARTICLES IMPACT ON COMMENSAL AND HOST AQUATIC DYNAMICS

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Abstract

In recent years, research trends are concentrated towards nanoparticles in many fields like aquatic ecosystems, clinical and pharma, agro and food chain-based industries. Nanomaterial plays a very crucial role in all the above fields, very particularly in medical treatments and experiments, aquatic ecosystems. The current study is to examine the ecological effects of nanoparticles in a commensal-host aquatic model with commensal species interference with the effect of nanoparticles. We examined the direct and indirect influence of these particles on the proposed model in view of stability of the system with appropriate chosen attributes. Our findings are more focused on nanoparticle-induced aquatic ecosystem which may stabilize or destabilize the system, resulting in sensitive analysis through various parametric values. We also found that as the contact rate between nanoparticles and the host increases, the equilibrium densities of the host and commensal fall. Furthermore, we notice that the depletion of nanoparticles from the aquatic system plays a vital role for the steady coexistence of both populations. Finally, the analytical results are verified and exposed through computer simulations which are quite interesting.

Keywords: Mathematical model, commensal, host, steady states, aquatic dynamics, nano-particle, stability.

Introduction

Mathematical modelling is a crucial multidisciplinary activity that entails the investigation of many aspects of several fields. Some of these subjects include biology, epidemiology, physiology, ecology, immunology, bio economics, and genetics. This mathematical modelling has risen to its pinnacle in recent years, spreading to all walks of life and attracting the attention of everybody. Lotka (1925) and Volterra (1931) were the pioneers of

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ecology mathematical modelling. Since then, other mathematicians and ecologists have contributed to the growth of this field of knowledge, including May (1973), Kushing (1979), Smith (1974), and Kapur (1985). Srinivas (1991) looked into competitive eco-systems with two and three species with restricted and infinite resources, whereas Laxminarayan and Ramacharyulu (2006) looked into Prey-Predator ecological models with partial cover for the prey and substitute food for the predator. Reddy (2009) and Sharma (2010) looked on a variety of issues involving two species competing systems with a temporal delay. Kumar (2010) went on to research several mathematical models of ecological commensalism. The conditions for a four-species syn eco-system were recently examined in depth by the authors Prasad and Ramacharyulu (2010, 2011, 2012). The current research is focused on a discrete model of two-species commensalism. The schematic sketch of the system under inquiry is shown in Figure 1.

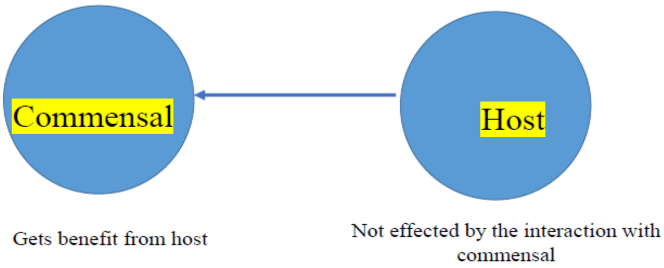

Figure 1. The schematic representation of the commensal-host system.

Nanoparticles have recently piqued the curiosity of scientists from a variety of fields, including biology, physics, chemistry, nanotechnology, and others. Although nanoparticles have a wide range of applications, they can sometimes cause harm to living organisms, including people and marine plankton. According to Kokate et al. (2011) human immune cells are susceptible to the cytotoxicity of zinc oxide nanoparticles. According to research by the Scientific Committee on Consumer Safety SCCS (2012), nano compounds have been discovered in cosmetics and sunscreens, but the health effects are yet unknown. In their study, Oszlánczi et al. (2011) discovered that diesel nanoparticles can affect the cardiovascular system of mice. On the other hand, Miglietta et al. (2011) looked at the ecotoxicological consequences of several types of nanoparticles on marine organisms. In recent years, many kinds of nanoparticles' potential toxicity to aquatic species like plants, fungi, algae, crustaceans, and fish have been described (for a full study (Pérez et al. (2009); Kahru et.al (2010); Stevenson et al. (2013); Castro-Bugallo et.al (2014); Das et al. (2014)). Nanoparticles are rapidly being found in a range of goods that end up in the aquatic environment (including drug delivery, cancer therapy, cosmetics, sun screens, clothing, paints, lithium-ion batteries, fertilizer, processed foods, and so on). This can occur directly (for example, as a result of an accident) or indirectly (for example, during manufacturing, use, or disposal) (through waste water). Natural processes such as forest fires, volcanic eruptions, weathering and creation of clay minerals, wind and water erosion, and desert dust storms can also release nanoparticles (Smita et al. (2012). These nanoparticles come in a wide range of sizes, and they can travel thousands of kilometers and remain suspended in the air for days (Smita (2012)), eventually settling in seawater. Because nanoparticles are eventually dissolved in seawater after being discharged, researchers must investigate their activity in seawater to determine whether they constitute a harm to living creatures. The dispersion and behavior of nanoparticles in aquatic systems define the majority of their interactions with marine animals, and the risk is typically tied to their surface speciation (Labille et al. (2010)). Internalization and/or attachment of nanoparticles to phytoplankton cells inhibit the growth of a wide range of phytoplankton species. Toxicity of silver, copper, aluminum, nickel, and cobalt nanoparticles in zebra fish, daphnids, and algal species was also investigated (Miao et al.(2010), Miller et al. (2012)). Titanium dioxide nanoparticles have been found to have a considerable detrimental effect on marine phytoplankton when exposed to normal amounts of UV exposure. In an experimental environment, Miller et al. (2012) studied the toxicity of nanoparticles in terms of population growth suppression. They used four common phytoplankton species (Isochrysis galbana, Thalassiosira pseudonana, Dunaliella tertiolecta, and Skeletonema costatum) to represent three major groups: diatoms (Bacillariophyceae), green algae (Chlorophyceae), and prymnesiophytes (Prymnesiophyceae) (Class: Prymnesiophyceae). The research used UV exposure (two levels: exposed and blocked) and TiO2 concentration (five levels: 0, 1, 3, 5, 7 mg). Srinivas et al. studied the effects of randomly changing driving factors on the proliferation of commensal and host species at time 't' in a standard eco system in 2013. The model is made up of commensal species and host species that benefit the commensal without being harmed in any way. A pair of non-linear differential equations characterize the model. The technique of Fourier transform is used to calculate stochastic stability in terms of the variances of the populations of the given system. Chattopadhyay et al. (2015) presented and investigated a simple modification of the Rosenzweig–MacArthur predator (zooplankton)-prey (phytoplankton) model with predator interference by including the influence of nanoparticles. They hypothesized that the influence of these particles could reduce the prey's maximal physiological per-capita growth rate. Nanoparticle dynamics are assumed to follow a straightforward Lotka-Volterra uptake term. Their findings imply that nanoparticle-induced phytoplankton population growth suppression can destabilize the system, resulting in limit cycle oscillation. They also discovered that as the interaction rate between nanoparticles and phytoplankton increases, phytoplankton and zooplankton equilibrium densities fall. Furthermore, they discovered that depletion/removal of nanoparticles from the aquatic environment is critical for both species' stable cohabitation. In compared to other widely utilized functional responses, their examination with several forms of functional responses reveals that the bedding ton functional response is the best accurate description of phytoplankton-nanoparticle interaction. In this article, we analyzed the impact of nano particles on stability of commensal host system by the motivation of (Srinivas et al. (2013), Rana et al. (2015)).

Mathematical Model

To investigate the effects of nanoparticles on a commensal-host system, we first use a simple commensal–host model. A system of ordinary differential equations is used to express the mathematical model [34], as shown below.

$$\left.\begin{aligned} x'(t) &= a_1 x - a_{11}x^2 + a_{12}xy \\ y'(t) &= a_2 y - a_{22}y^2 \end{aligned}\right\} \tag{1}$$

Here x denotes the density of commensal population and y denotes the density of host populations at time t. In the absence of commensal, the host population follows the logistic growth with an intrinsic growth rate r_1 and carrying capacity K_1. Commensal commensurate the host (Srinivas et al. (2013)) with commensal rate c, the natural death rate of host is d. We also believe that when nanoparticles in the aquatic system come into close contact with commensal, they attach to the population of the commensal and occasionally enter the host. Free nanoparticles are eliminated from the aquatic system due to attachment of nanoparticles in commensal cells. Following the mathematical modelling of free virus internalization into bacteria (Beretta et al. (2000)), we integrate the internalization of nanoparticles with the commensal and host species into the model system. With the aforementioned assumptions in mind, we extend the previous model in the presence of nanoparticles as follows:

$$\left.\begin{aligned} x'(t) &= r_1 x\big(1-(x/K_1)\big)/(1+\alpha_1\alpha_2 N_\alpha x)+cxy \\ y'(t) &= r_2 y - r_3 y^2 - dy \\ N'_\alpha(t) &= A-\alpha_1 N_\alpha x - d_1 N_\alpha \end{aligned}\right\} \tag{2}$$

All parameters are expected to be positive in this case. It's worth noting that, in the absence of nanoparticles, model (2) becomes model (1).

Boundedness

Theorem 3.1: All solutions of (2) which originate in $\mathbb{R}_+^3$ are uniformly bounded.

Proof: From the third equation of (2), we have $N'_\alpha(t) \le A - d_1 N_\alpha$

i.e. $N'_\alpha(t) - d_1 N_\alpha \le A$, Therefore, $0 < N_\alpha(t) \le e^{-d_1 t}\left(N_\alpha(0)-(A/d_1)\right)+(A/d_1)$

As $t\to\infty, N_\alpha(t)\to A/d_1$, since sup $N_\alpha(t)=A/d_1$

Let us define a function $W = r_2 x + y$ and let us calculate the time derivative of W , we obtain

$$W'(t) \le r_2 r_1 x\big(1-(x/K_1)\big) - dy$$

$$W'(t)+\eta W \le x\big(r_1 r_2 + \eta r_2 - (r_1 r_2 x/K_1)\big) + \big(\eta - d_1\big)y$$

$$W'(t)+\eta W \le m$$

Here $\eta < d_1$, $m = r_2 K_1 (r_1+\eta)^2/4r_1$; Therefore, $0 < W(t) \le e^{-\eta t}\left(W(0)-\eta^{-1}\right)+\eta^{-1}$

As $t\to\infty, W(t)\to m/\eta$, i.e. $W(t)\le m/\eta$, since sup $W(t) = m/d_1$. As a result, system (2) is constrained above. This suggests that during a long-time interval, none of the interacting species grows exponentially. Due to limited resources, the number of each species is limited.

Equilibrium analysis

The system has four steady states as $E_0\left(0,0,0\right)$, $E_1\left(0,0,A/d_1\right)$, $E_2\left(K_1,0,A_1/(\alpha_1 K_1+d_1)\right)$, $E_4\left(x^*,y^*,N_\alpha^*\right)$, where $x^* = K_1(r_1 r_3 + c(r_2-d))/(r_1 r_3 - K_1 c\alpha_1\alpha_2 N_\alpha(r_2-d))$; $y^* = (r_2-d)/r_3$; $N_\alpha^* = A/(\alpha_1 K_1 + d_1)$. For y^* to be positive, we must have, $r_2 > d$ and for x^* to be positive, we must have $r_1 r_3 > K_1 c\alpha_1\alpha_2 N_\alpha (r_2-d)$

Local stability

Lemma 5.1: The system (2) around $E_4\left(x^*,y^*,N_\alpha^*\right)$ is locally asymptotically stable if the root of the characteristic equation $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$ of the Jacobian matrix $J(E^*)$ satisfies Routh–Hurwitz criteria i.e., $A_1 > 0, A_3 > 0, A_1 A_2 - A_3 > 0$ where

$$J(E^*) = \begin{bmatrix} J_{11}-\lambda & J_{12} & J_{13} \\ J_{21} & J_{22}-\lambda & J_{23} \\ J_{31} & J_{32} & J_{33}-\lambda \end{bmatrix}$$

where $J_{11} = -cy^*\left[K_1 - 2x^* - x^*\alpha_1\alpha_2 N_\alpha^*\right]/K_1 - x^*$; $J_{12} = cx^*$; $J_{13} = 0$; $J_{21} = 0$; $J_{22} = -r_3 y^*$

$J_{23} = 0$; $J_{31} = -\alpha_1 N_\alpha^*$; $J_{32} = 0$; $J_{33} = -\alpha_1 x^* - d_1$, and $A_1 = J_{22} + J_{33} - J_{11}$; $A_3 = -J_{11}J_{22}J_{33}$

$A_2 = J_{11}J_{22} + J_{11}J_{33} + J_{22}J_{33}$; For $A_1 > 0, A_3 > 0, A_1A_2 - A_3 > 0$, we must have $K_1/(2+\alpha_1\alpha_2 N_\alpha^*) < x^* < K_1$ and $x^* = \min\left\{K_1, K_1/(2+\alpha_1\alpha_2 N_\alpha^*)\right\}$

Global stability

In this section, we investigated the global stability behavior of the system (2) at the interior equilibrium $E_4\left(x^*, y^*, N_\alpha^*\right)$ by using the Lyapunov stability theorem.

Theorem: The system (2) is globally asymptotically stable about $E_4\left(x^*, y^*, N_\alpha^*\right)$

Proof: Let us consider the Lyapunov function

$$V\left(x,y,N_\alpha\right) = \left(x - x^* - \ln\left(x/x^*\right)\right) + l_1\left(y - y^* - \ln\left(y/y^*\right)\right) + l_2\left(N_\alpha - N_\alpha^* - \ln\left(N_\alpha/N_\alpha^*\right)\right)$$

Differentiating $V\left(x,y,N_\alpha\right)$ with respect to ‘t’ we get,

$$V'(t) = l_1\left(x - x^*\right)x'(t) + l_2\left(y - y^*\right)y'(t) + l_3\left(N_\alpha - N_\alpha^*\right)N_\alpha'(t)$$

where l_1, l_2, l_3 are positive constants.

$$V'(t) = l_1\left(x - x^*\right)\left[\frac{r_1}{1+\alpha_1\alpha_2 N_\alpha x}\left(1 - \frac{x}{K_1}\right) + cy\right] + l_2\left(y - y^*\right)\left[r_2 - r_3 y - d\right] + l_3\left(N_\alpha - N_\alpha^*\right)\left[(A/N_\alpha) - \alpha_1 x - d_1\right]$$

$$V'(t) = \left(x - x^*\right)\left[\frac{-r_1\left(x - x^*\right)}{K_1\left(1+\alpha_1\alpha_2 N_\alpha x\right)\left(1+\alpha_1\alpha_2 N_\alpha x^*\right)} - \frac{r_1\alpha_1\alpha_2 N_\alpha^*\left(x - x^*\right)}{K_1\left(1+\alpha_1\alpha_2 N_\alpha x\right)\left(1+\alpha_1\alpha_2 N_\alpha x^*\right)}\right] + l_1\left(y - y^*\right)\left[r_2 y^* - r_3 y\right] + l_2\left(N_\alpha - N_\alpha^*\right)\left[d_1 N_\alpha^* - d_1 N_\alpha\right]$$

By choosing $l_1 = 1/r_3; l_2 = N_\alpha/d_1$,

$$V'(t) = -\frac{r_1\left(x - x^*\right)^2}{K_1\left(1+\alpha_1\alpha_2 N_\alpha x\right)\left(1+\alpha_1\alpha_2 N_\alpha x^*\right)} - \frac{r_1\alpha_1\alpha_2 N_\alpha^*\left(x - x^*\right)^2}{\left(1+\alpha_1\alpha_2 N_\alpha x\right)\left(1+\alpha_1\alpha_2 N_\alpha x^*\right)}$$
$$-\left(y - y^*\right)^2 - \left(N_\alpha - N_\alpha^*\right)^2$$

$V'(t) < 0$. Therefore, the system (2) is globally asymptotically stable about $E_4\left(x^*, y^*, N_\alpha^*\right)$

Numerical analysis

In order to investigate the dynamics of the model, we perform numerical simulations of the model as a standard initial value problem (2), incorporating a relevant initial condition. For this, we take the parameter values as follows.

Example 1: For the parameter values

$$r_1 = 1.5; r_2 = 2.5; r_3 = 1.5; \alpha_1 = 0.1; \alpha_2 = 0.8; K_1 = 20; A = 5; d = 0.08, d_1 = 0.05; c = 0.8$$

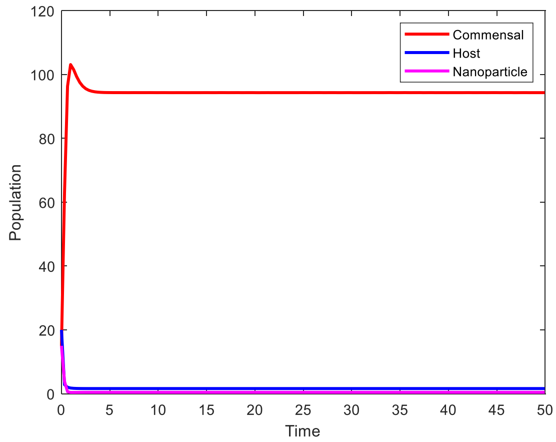

Figure 2. Time series evaluation of population with the attributes of example-1.

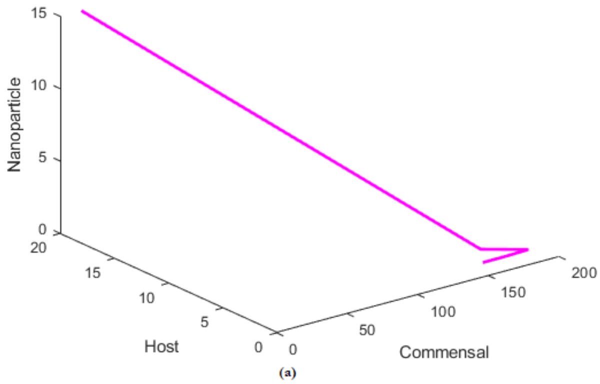

Figure 3. Phase portrait of commensal, host and nano particles with the attributes of example-1.

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Example 2: For the parameter values

$$r_1 = 1.8; r_2 = 3.5; r_3 = 2.5; \alpha_1 = 0.12; \alpha_2 = 0.8; K_1 = 20; A = 5; d = 0.08, d_1 = 0.05; c = 1.8$$

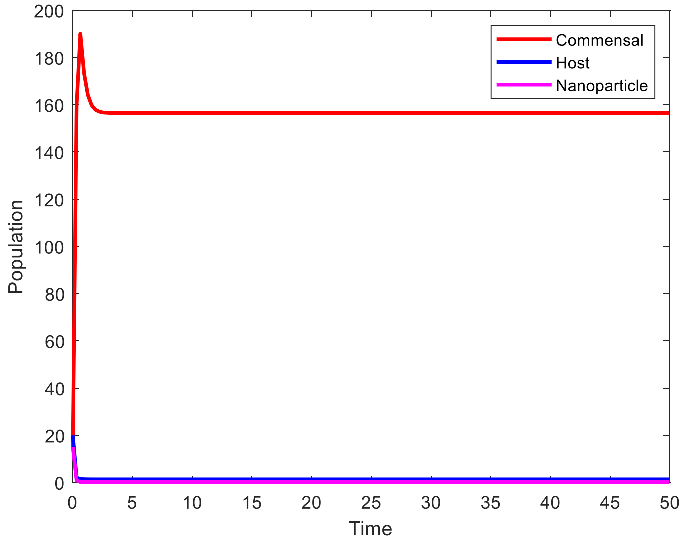


Figure 4. Time series evaluation of population with the attributes of example-2.

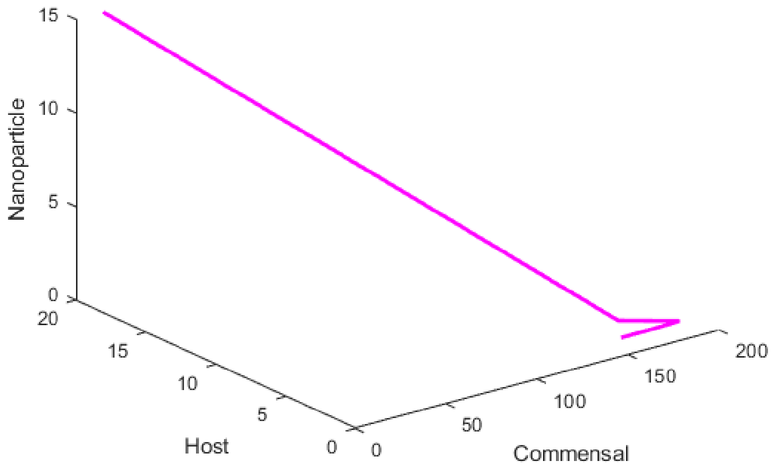
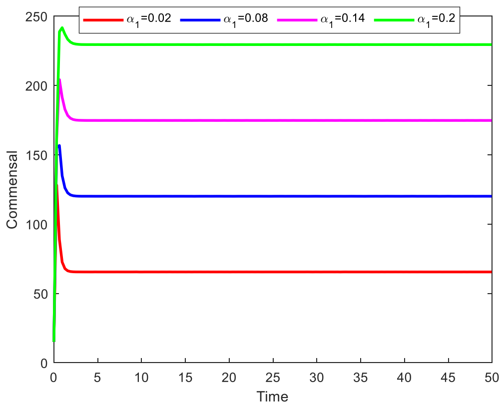


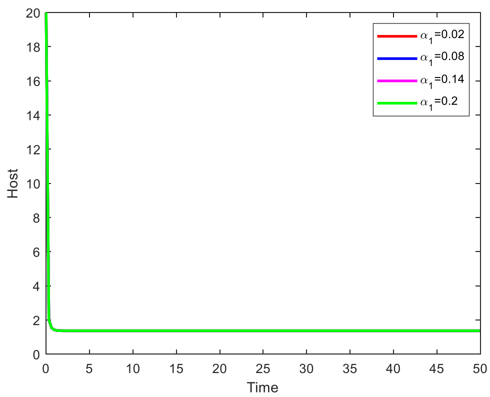
Figure 5. Phase portrait of commensal, host and nano particles with the attributes of example-2.

Example 3: For the parameters,

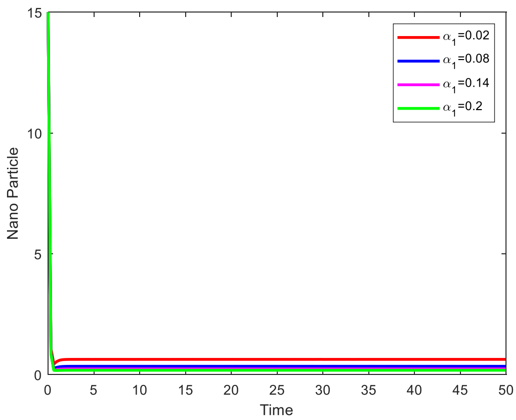
r_1=1.8; r_2=3.5; α_2=0.8; K_1=20; K_2=20;A=5;d_1=0.05;d=0.08;c=1.8; α_1=0.12;r_3=2.5;



(a)



(b)



(c)

Figures 6(a),2(b),2(c). The time series evaluation of Commensal, Host, and Nano Particle for various values of alpha1 along with the values of Example 3.

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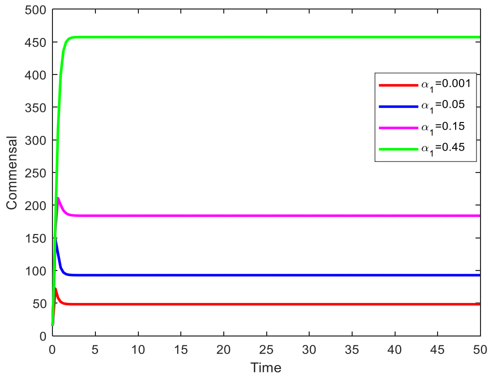
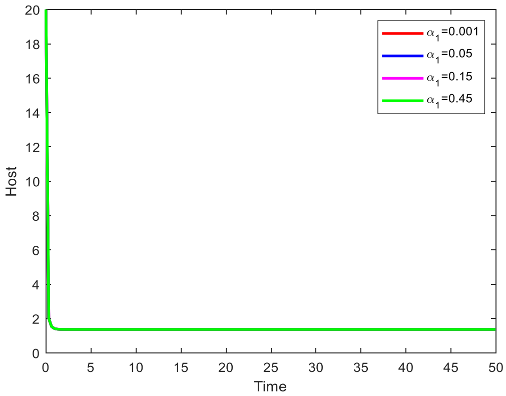


(a)



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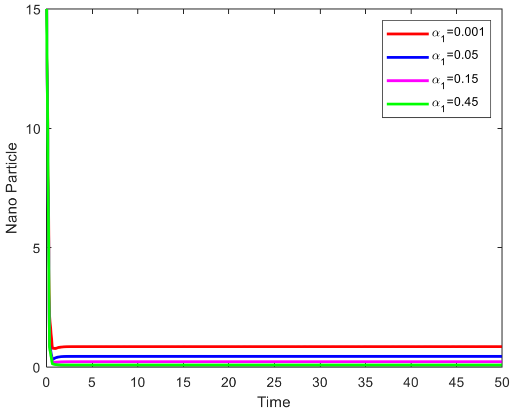
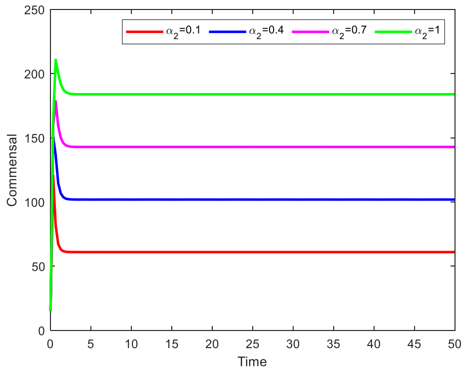
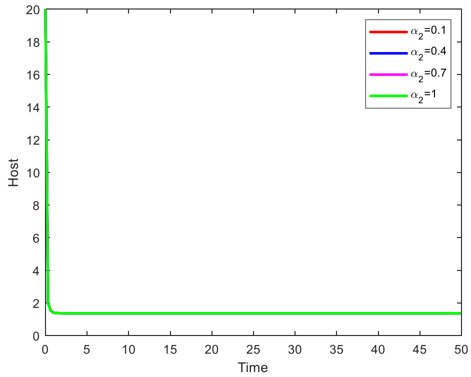


(c)

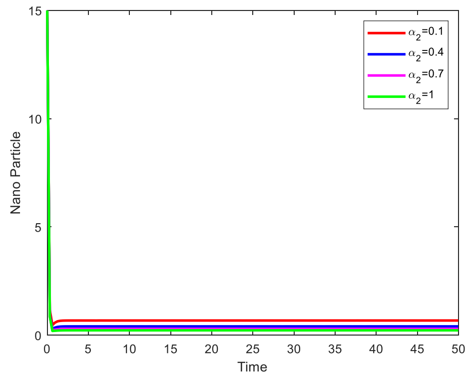
Figures 7(a),3(b),3(c). The time series evaluation of Commensal, Host, and Nano Particle for various values of alpha1 along with the values of Example 3.



(a)



(b)



(c)

Figures 8(a),4(b),4(c). The time series evaluation of Commensal, Host, and Nano Particle for various values of alpha2 along with the values of Example 3.

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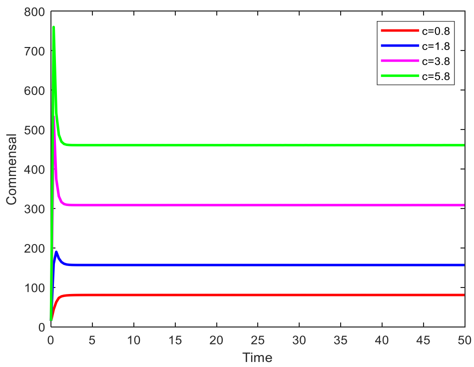
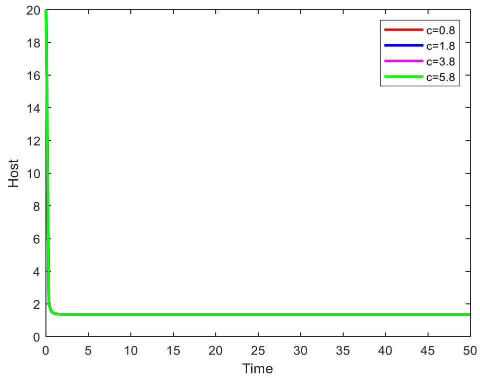
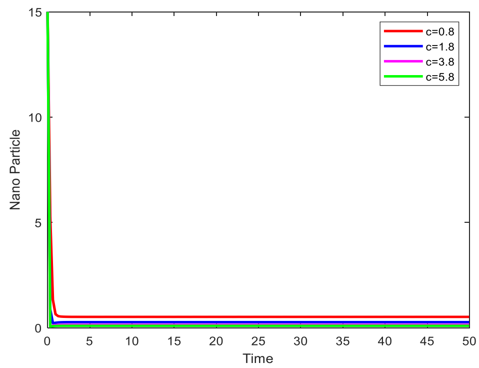


(a)



(b)



(c)

Figures 9(a),5(b),5(c). The time series evaluation of Commensal, Host, and Nano Particle for various values of 'c' along with the values of Example 3.

Effect of Nanoparticles on Commensal and Host Population

The influence of Nanoparticles on the aquatic food chain, as exemplified by the commensal-host interaction, is of particular importance. As a result, the crucial parameter is the interaction rate between commensal and nanoparticles or α_1 . More Nanoparticles will affect the oceanic food web if the contact rate α_1 is larger. We investigate the change in the equilibrium density of commensal and host to determine the sole mechanism following the introduction of Nanoparticles into the system. It should be highlighted that when the host and commensal interact normally, nanoparticles have no influence on commensal equilibrium densities, despite the fact that experimental evidence demonstrates that nanoparticles suppress commensal growth and diminish cell density. Figure 6a, 2b and 2c shows the variations of α_1 on Commensal, Host and Nano Particle. Commensal populace increasing as α_1 increasing is shown very clearly in Figure 6a. Host Populace is not affected by variations in α_1 is visible in Figure 2b. Nano Particles slightly affected (almost negligible) for increasing values of α_1 is represented by Figure 2c. Finally, we can conclude that Nanoparticles association with species effects commensal populace greatly where as host populace doesn't affect by the presence of Nanoparticles. Figure 7a, 7b and 7c shows the variations of α_1 on Commensal, Host and Nano Particle. Commensal populace increasing as α_1 increasing is shown very clearly in Figure 7a. Host Populace is not affected by variations in α_1 is visible in Figure 7b. Nano Particles little affected for increasing values of α_1 is represented by Figure 7c. Finally, we can conclude that Nanoparticles association with species effects commensal populace greatly where as host populace doesn't affect by the presence of Nanoparticles. Figures 6a, 6b, and 6c, Figures 7a, 7b and 7c are varied in alpha1 values which are slight and more differences in values respectively. Figure 8a, 8b and 8c shows the variations of α_2 on Commensal, Host and Nano Particle. Commensal populace increasing as α_2 increasing is shown very clearly in Figure 8a. Host Populace is not affected by variations in α_2 is visible in Figure 8b. Nano Particles slightly affected (almost negligible) for increasing values of α_2 is represented by Figure 8c. Finally, we can conclude that Nanoparticles association with species effects commensal populace greatly where as host populace doesn't affect by the presence of Nanoparticles. Figure 9a, 9b and 9c shows the variations of c on Commensal, Host and Nano Particle. Commensal populace increasing as c increasing is shown very clearly in Figure 9a. Host Populace is not affected by variations in c is visible in Figure 9b. Nano Particles slightly affected (almost emerging) for increasing values of c is represented by Figure 9c. Finally, we can conclude that Commensal commensurate the host with commensal rate c effects commensal populace greatly where as host populace doesn't affect by the variations in the value of c

Concluding remarks

In this article, we proposed commensal-host-Nanoparticle model. We studied and investigated the steady states, and local, global stabilities around interior equilibrium point. It is also verified the boundedness. Finally, we conclude that the effect of nanoparticles on commensal population with the crucial parameter α_1 which is the contact rate of commensal and host. Similarly, it is verified the effect of commensal and host population with α_2 and c also. Finally, we can conclude that the presence of Nano particle affects greatly the commensal populace even the parameter analysis in the form of graphs also exhibits the same. Based on the nature and properties of Nano particle the effects on commensal and host are more notable, which may be the future scope of this work. With this work, we can conclude the Nanoparticle affects the populace and system to certain extent in the form of increasing population densities. Based on the physical and chemical properties of Nano particles which are present in the aquatic eco system. The results further can be deeply analyzed by experimental tools and techniques, may be one of our future projects.

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