Abstract: Violation of matter-antimatter symmetry, the so-called CP violation, was observed only as a small effect in the decays of the neutral K meson in 1964. In order to study the long-standing puzzle about the origin of the CP violation, the KEK B-factory has been constructed at High Energy Accelerator Research Organization (KEK) in Japan. Using the KEK B-factory and the BELLE detector, an experimental study of the production and decay of the B meson will be carried out to search for the CP violation. Using the BELLE Fast Simulator (FSIM), we made a Monte Carlo simulation study to estimate an integrated luminosity required to observe the signal and CP violation in the charmless hadronic decays of the B mesons. The sensitivity of the BELLE experiment to the measurement of the direct CP violation and branching ratio has been estimated.

Keywords: CP violation; B-factory experiment; Charmless decay; Monte Carlo Simulation; Branching ratio
study the origin of the large branching ratio of this decay mode. Motivated by the measurement of
the large branching ratio of $B^\pm \eta' K^\pm$ many theoretical studies have been made to explain the
discrepancy between the standard theory and the experiment (Du and Guo, 1997, Cheng and Tseng,
clear whether any of these theoretical calculations can explain the experimental data or not because
of the large uncertainties in the theoretical calculations and the large statistical errors of the
experimental data. In order to understand the origin of the large branching ratio, therefore, it is
important to measure the branching ratio of the $B^\pm \eta' K^\pm$ decay with an accuracy of about 10 %
statistical error.

In this paper, we report results of a Monte-carlo simulation study based on a fast detector simulator
(FSIM) on the $B^\pm \rightarrow \eta' K^\pm$, $\eta' \rightarrow \eta \pi^+ \pi^-$ followed by $\eta \rightarrow \gamma\gamma$ decay mode for the BELLE experiment at
KEK B-Factory. Through the present simulation study, we estimate an integrated luminosity
required to observe signals and CP violation with a given significance in this decay mode.

Materials and Methods

**KEK B-Factory and the BELLE Detector:** Detailed studies of the B physics require a large
number of B mesons because of its small branching ratios in the final states. The B-factory at KEK
in Japan, referred as “KEKB”, is an asymmetric electron-positron collider of energies $8.0 \times 3.5$ GeV,
which aims to provide electron-positron collision at a center of mass energy 10.58 GeV that
corresponds to $\Upsilon(4S)$ resonance. The $\Upsilon(4S)$ is the first resonance above the B meson production
threshold, decaying to a pair of $B^+ B^-$ or $B^0 \bar{B}^0$ in a approximately equal decay rate. The primary
goal of the BELLE experiment is to detect the CP violating effects in B meson decays and to
provide a definitive information regarding the mechanism of the CP violation (BELLE
Collaboration, 1994).

Fig. 1. Side view of the BELLE detector.
Figure 1 shows the side view of the BELLE detector which consists of several sub-detectors. Decay vertices of the B mesons are reconstructed by a Silicon Vertex Detector (SVD). Charged particle tracking is provided by a Central Drift Chamber (CDC) that is co-axial with the beam. Particle identification is provided by the dE/dx measurement in the CDC, the Aerogel Cherenkov Counter (ACC) and the time of flight (TOF) counter arrays. The TOF and ACC are located radially outside of the CDC. Electromagnetic showers are detected in a nine-thousand block array of CsI(Tl) crystals located inside the solenoid coil. Muons and K_L are identified by arrays of detectors interspersed in iron return yoke of magnet. The details of this detector can be obtained from any of the KEK memoranda (BELLE Collaboration, 1994; Khan, 1999; Suda, 1998).

**Event Generation:** The two body charmless hadronic decay events were generated using QQ event generator (Itho, 1995) in which the latest experimental results from CLEO-II detector are installed. By using QQ generator, the following two data sets corresponding to 100 fb^{-1} were prepared for the present study:

- The first data set consists of e^+e^- → \Upsilon(4S) → B^+B^- events one of which decays as B^\pm → η'K^\pm, η' → ηπ^+π^- followed by η → γγ, whereas the other partner B decays in the standard decay mode. This data set contains 1340 generated events.
- The second data set contains continuum background events which do not form the \Upsilon(4S) resonance.

These data sets are subsequently passed through the BELLE fast simulator (FSIM) version 5.2 (Ozaki, 1996) using the latest information on the acceptance and performances of the BELLE detector. The information from the CDC, ACC, TOF and ECL subdetectors were used in the present analysis. We transform all the measured quantities (e.g. track momentum, energy, etc.) into those in the CM frame of the B^+B^- pair first and then analysis was performed. The parameters used in the present analysis are shown in Table 1.

### Table 1. The parameters used in the present analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The cross-section for the production of the \Upsilon(4S) resonance</td>
<td>1.2 nb*</td>
</tr>
<tr>
<td>The cross-section for the production of a QQ pair</td>
<td>3.5 nb</td>
</tr>
<tr>
<td>Integrated luminosity</td>
<td>100 fb^{-1}</td>
</tr>
<tr>
<td>Br(B^\pm→η'K^\pm)</td>
<td>6.5×10^{-5}**</td>
</tr>
<tr>
<td>Br(η'→ηπ^+π^-)</td>
<td>0.437</td>
</tr>
<tr>
<td>Br(η→γγ)</td>
<td>0.3925</td>
</tr>
</tbody>
</table>

* We assume that \Upsilon(4S) decays into B^+B^- and B^0B^0 with equal probability. ** Behrens, 1998.

**General Description of the Selection Procedure:** The B candidate was reconstructed both from the \Upsilon(4S) events and from the continuum background events. To select the B candidate for B^\pm→η'K^\pm the following cuts and selections are used.

**a. The beam constrained mass (M_b):** The invariant mass M of the B candidate was derived using the relation

\[ M_B = \sqrt{(E_B^2 - P_B^2)} \]  \hspace{1cm} \text{(1)}

where \( P_B \) and \( E_B \) are the measured momentum and energy of the B candidate. Using the beam energy \( E_{\text{beam}} \) that is accurately known instead of measured energy \( E_B \) of the B candidate the mass resolution can be improved by the order of magnitude. Therefore Eq.(1) can be replaced as

\[ M_b = \sqrt{(E_{\text{beam}}^2 - P_B^2)} \]  \hspace{1cm} \text{(2)}

\[ (1) \]
b. Energy Constrained ($\Delta E$): Conservation of energy requires that the $B$ meson and their decay products should carry the full energy of the beam. We define $\Delta E$ as

$$\Delta E = E_1 + E_2 - E_{\text{beam}}$$  \hspace{1cm} (3)$$

where $E_1$ and $E_2$ are energy of the $\eta'$ and $K^\pm$, respectively. For true $B$ candidate the $\Delta E$ should have a peak at zero whereas for the continuum background it should have uniform distribution. The true $B$ candidates were separated from the continuum background events using the $\Delta E$ distribution in this analysis.

c. Thrust angle cut: The dominant background comes from the continuum hadronic production, i.e. from $e^+e^-\rightarrow q\bar{q}$, where $q$ denotes $u,d,s$ and $c$ quarks. Since the produced $B$ meson from $\Upsilon(4S)$ has kinetic energy of a few MeV, they are approximately at rest in the CMS. The decay products of $B$ mesons are thus spherically distributed and are not correlated with each other. On the other hand the hadrons from the continuum $qq$ production tend to appear as a two-jet like structure. To distinguish the continuum events from the signal events, we calculated the angle $\theta_T$ between the thrust axis of the $B$ candidate particle and that of all the remaining charged and neutral particles in the event.

The thrust axis is defined as a line along the vector $\mathbf{n}$ which gives the maximum of

$$T (\mathbf{n}) = \frac{\sum_i | \mathbf{P}_i \cdot \mathbf{n} |}{\sum_i | \mathbf{P}_i |}$$  \hspace{1cm} (4)$$

where $\mathbf{P}_i$ is the momentum vector of $i$th particle among the $B$ candidate ones or the remaining ones in the event. Since the continuum $qq$ events form a two-jet structure and $B^+B^-$ events have no axis correlation, the distribution of $\cos \theta_T$ is strongly peaked near $\cos \theta_T = \pm 1$ for $qq$ events and nearly flat for $B^+B^-$ events. This is clearly seen in Fig. 2 which shows the $\cos \theta_T$ distribution (a) for the $e^+e^-\rightarrow qq$ and (b) for $e^+e^-\rightarrow \Upsilon(4S) \rightarrow B^+B^-$ events normalized to the same integrated luminosity.

![Fig. 2. The $\cos \theta_T$ distribution (a) for the $e^+e^-\rightarrow qq$ continuum background and (b) for $e^+e^-\rightarrow \Upsilon(4S) \rightarrow B^+B^-$ signal events normalized to the same integrated luminosity.](image-url)
d. Particle Identification: In the present analysis the subroutines FCDCPID, FAERPID and TOFPID were used in FSIM for the purpose of charged particle identification using the information from the CDC, ACC and TOF sub-detectors, respectively. In the routines FCDCPID and FTOFPID, the probability of a track being a particular type (= e, µ, π, K, p) is calculated. The combined probability, prob\(^i\), is obtained as

\[ \text{prob}\(^i\) = \text{prob(CDC)}\(^i\). \text{prob(ACC)}\(^i\). \text{prob(TOF)}\(^i\) \] ………………..(5)

In the present study separation of a kaon from a pion was carried out by using the PID cut requiring

\[ \text{prob}_K \geq \text{prob}_\pi \] ………………………………..……............…......(6)

Reconstruction Procedure of the B candidate for \( B^\pm \rightarrow \eta'K^\pm \), \( \eta' \rightarrow \eta\pi^\pm \pi^- \) followed by \( \eta \rightarrow \gamma\gamma \):

For the reconstruction of \( \eta \) meson, all the possible 2\(^\gamma\) pairs were combined. Fig. 3 shows the invariant mass distribution of the two gamma system which has a \( \pi^0 \) peak (0.135 GeV) as well as \( \eta \) one (0.55 GeV). The \( \pi^0 \) peak at 0.135 GeV is expected to come from the standard decay of \( \bar{B} \).

![Graph showing invariant mass distribution](image)

The \( \eta \) meson was selected applying a 3\(\sigma\) cut around the \( \eta \) mass peak (0.5186 GeV\( \leq m_{\gamma\gamma} \leq 0.5786 \) GeV). Then the selected \( \eta \) meson is combined with two charged tracks with opposite charges to reconstruct \( \eta' \) meson. All the possible charged particles were combined with the \( \eta \) meson assuming that they are pions. Fig. 4(a) shows the invariant mass distribution of \( \eta\pi^\pm \pi^- \). The peak around the mass of \( \eta' \) (0.958 GeV) is seen but it is wider compared to the natural width of \( \eta' \). To make \( \eta' \) mass resolution much more precise, the mass constrained fit on \( \eta \) invariant mass was used. The invariant mass distribution for the \( \eta\pi^\pm \pi^- \) system after the mass constrained fit on \( \eta \) is shown in Fig. 4(b). The \( \eta' \) meson was selected by imposing a 3\(\sigma\) cut around the \( \eta' \) peak (0.948 GeV\( \leq m_{\eta\pi^\pm \pi^-} \leq 0.965 \) GeV).

Finally to reconstruct \( B \), the reconstructed \( \eta' \) was combined with another charged track assuming that it is a kaon. Using the measured energies and momenta of \( \eta' \) and K, the energy imbalance \( \Delta E \) and the beam constrained mass \( M_b \) were obtained. Since the mass resolution of the \( B \) invariant
mass is worse than that of the beam constrained mass $M_b$ we used the beam constrained mass $M_b$, instead of B invariant mass $M_B$ in the present analysis.

Finally to reconstruct B, the reconstructed $\eta'$ was combined with another charged track assuming that it is a kaon. Using the measured energies and momenta of $\eta'$ and K, the energy imbalance $\Delta E$ and the beam constrained mass $M_b$ were obtained. Since the mass resolution of the B invariant mass is worse than that of the beam constrained mass $M_b$ we used the beam constrained mass $M_b$, instead of B invariant mass $M_B$ in the present analysis.

Applying a $\Delta E$ cut of $|\Delta E| \leq 0.04$ GeV on this $\Delta E$ distribution the S/N (signal/noise) ratio was calculated to be 0.4 with a reconstruction efficiency $\varepsilon$ of 46.9%. The reconstruction efficiency $\varepsilon$ is defined as the ratio of the measured number of signal events to the number of signal events generated. In order to eliminate the continuum background, the $M_b+\Delta E$ cut, $M_b+\text{PID}+\Delta E$ cut, $M_b+$thrust angle+ $\Delta E$ cut and $M_b+\text{PID}+$thrust angle+$\Delta E$ cut were used in the present analysis successively to select the best cut. Figs. 5(a) and 5(b) show the $\Delta E$ distributions after the $M_b+\text{PID}$ cut and the $M_b+\text{PID}+$thrust angle cut, respectively. It is shown that the highest sensitivity can be achieved with the $M_b+\text{PID}+$thrust angle cut with a cut value of $|\cos \theta_T| \leq 0.9$ when a $\Delta E$ cut of $|\Delta E| \leq 0.04$ GeV is used in Fig.5(b). The statistical significance of the signal after this cut is $6.2\sigma$ at an integrated luminosity of 10 fb$^{-1}$. With this condition the S/N ratio is 2.5 and the reconstruction efficiency is 40.4%. In this study, it has been found that the PID cut reduces the continuum background events down to $\sim 40\%$ level. The S/N ratio, reconstruction efficiency and statistical significance of the signal after several different combinations of cuts are summarized in Table 2. It is clear that the $M_b+\text{PID}+$thrust angle+$\Delta E$ cut is the optimized cut for the present analysis.

Table 2. The number of the signal events S, the background events N, the S/N ratio, reconstruction efficiency $\varepsilon$ and significance of the signal $\Sigma_{signal}$ for different combinations of cuts.

<table>
<thead>
<tr>
<th>Cut</th>
<th>S</th>
<th>N</th>
<th>S/N</th>
<th>$\varepsilon$ (%)</th>
<th>$\Sigma_{signal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_b+\Delta E$</td>
<td>62.9±2.5</td>
<td>142.5±3.8</td>
<td>0.4±0.0</td>
<td>46.9±2.0</td>
<td>4.4±0.2</td>
</tr>
<tr>
<td>$M_b+\text{PID}+\Delta E$</td>
<td>58.8±2.4</td>
<td>48.0±2.2</td>
<td>1.2±0.1</td>
<td>43.9±2.0</td>
<td>5.7±0.2</td>
</tr>
<tr>
<td>$M_b+$thrust angle+$\Delta E$</td>
<td>45.3±2.1</td>
<td>19.3±1.4</td>
<td>2.3±0.2</td>
<td>33.8±2.0</td>
<td>5.6±0.2</td>
</tr>
<tr>
<td>$M_b+\text{PID}+$thrust angle+$\Delta E$</td>
<td>54.1±2.3</td>
<td>21.6±1.5</td>
<td>2.5±0.2</td>
<td>40.4±2.0</td>
<td>6.2±0.3</td>
</tr>
</tbody>
</table>
Results and Discussions

The data shown in Table 2 were used to estimate the required integrated luminosity to measure the branching ratio with a given statistical error and to observe a given CP asymmetry with a given significance. Results of these estimations are given in this section.

In order to compare the theoretical predictions with the experimental data, experimental studies with a much smaller relative statistical error than that of the presently available experimental data are important. At present, the branching ratios for the $B^\pm \rightarrow \eta^'K^\pm$ decay mode measured by CLEO collaboration has a large relative statistical error of about 23%. Through the present simulation study, a relative statistical error and a signal sensitivity at a given integrated luminosity were estimated for the decay mode $B^\pm \rightarrow \eta^'K^\pm$.

The total number of the $B^\pm \rightarrow \eta^'K^\pm$ events, $S$ is expressed in terms of the branching ratio and the integrated luminosity as

$$S = \sigma \times \int L dt \times <Br> \times Br_{sub} \times \varepsilon \hspace{1cm} (7)$$

where $\sigma$ is the cross-section for $\Upsilon(4S)$ production, $\int L dt$ the integrated luminosity, $<Br>$ is the average branching ratio of the $B^\pm$ decay, $Br_{sub}$ is the branching ratio for the sub-decay modes and $\varepsilon$ is the reconstruction efficiency. In the present simulation study, the number of the signal events $S$ was estimated independently from the continuum backgrounds. When the contribution from errors and the continuum background events to the measurements of the branching ratio $<Br>$ is taken into account, the relative statistical error of $<Br>$ is given by

$$\frac{\Delta <Br>}{<Br>} = \sqrt{\frac{(S + N)}{S}} \hspace{1cm} (8)$$

where $N$ is the number of the continuum background events. Finally the relative statistical error of the branching ratio in terms of the integrated luminosity is given as

$$\frac{\Delta <Br>}{<Br>} = \sqrt{\frac{1 + \frac{N}{S}}{\sigma \times \int L dt \times <Br> \times Br_{sub} \times \varepsilon}} \hspace{1cm} (9)$$
From Eq. (9) and the present simulation data, the relative statistical error of the branching ratio measurement at a given integrated luminosity was estimated. It has been shown that one can measure the $<\text{Br}>$ with relative statistical errors of $11.2\%$ for the present decay mode at an integrated luminosity of $10$ fb$^{-1}$. Data with this level of relative statistical error will give important information for the comparison of the experimental data with the theoretically predicted values. The required integrated luminosity to observe a given CP asymmetry can be expressed as

$$\int L dt = \frac{\varepsilon \times Br(1 + \frac{N}{S})}{\sigma(\varepsilon \times Br)^{2}} \times \frac{\sum_{\text{asym}}^{2}}{A_{\text{asym}}} \times (10)$$

Details of this derivation can be obtained from ref. (Khan, 1999). Using the present simulation data given in Table 2 and the measured branching ratio by CLEO, the required integrated luminosity to observe a given CP asymmetry with a given statistical significance $\sum_{\text{asym}}$ was calculated. In Table 3 the required integrated luminosity to observe $10\%$, $20\%$ and $40\%$ CP asymmetry for $2\sigma$ and $3\sigma$ significance is shown.

Table 3. Required integrated luminosity to observe a given CP asymmetry with for $2\sigma$ and $3\sigma$ significance of the asymmetry for $B^{\pm} \rightarrow \eta^{'}K^{\pm}$, $\eta^{'} \rightarrow \eta\pi^{+}\pi^{-}$ followed by $\eta \rightarrow \gamma\gamma$ decay mode.

<table>
<thead>
<tr>
<th>Asymmetry(%)</th>
<th>$2\sigma$</th>
<th>$3\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>6.9</td>
<td>15.4</td>
</tr>
<tr>
<td>20</td>
<td>27.4</td>
<td>60.6</td>
</tr>
<tr>
<td>10</td>
<td>109.6</td>
<td>246.5</td>
</tr>
</tbody>
</table>

Conclusion

In order to study a feasibility of the measurement of direct CP violation and of the branching ratio with much better accuracy than the presently available data, a Monte Carlo simulation study using a fast simulator has been carried out for two body charmless decays of B mesons ($B^{\pm} \rightarrow \eta^{'}K^{\pm}$). It has been found from the present simple cut analysis method that BELLE detector can detect the process $B^{\pm} \rightarrow \eta^{'}K^{\pm}$ with an efficiency of $40.4\%$. If the CP asymmetry is as large as $20\%$, $3\sigma$ asymmetry can be observed at an integrated luminosity of $60.6$ fb$^{-1}$. The relative statistical error at an integrated luminosity of $10$ fb$^{-1}$ is estimated to be $11.2\%$. The measurement of the branching ratio with the relative statistical error of this level will give an useful information to understand the origin of the large branching ratio.

References


