Image Reconstruction via Compressed Sensing

Raghib Shahriar
Electronics and Communication Engineering Discipline
Khulna University
Khulna-9208, Bangladesh
shahraghib.ece16@gmail.com

Nawshin Jahan Mowri
Electronics and Communication Engineering Discipline
Khulna University
Khulna-9208, Bangladesh
nawshinjahan57@gmail.com

Mohammad Ismat Kadir
Electronics and Communication Engineering Discipline
Khulna University
Khulna-9208, Bangladesh
ismat.kadir@ece.ku.ac.bd

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***Abstract*— Compressed Sensing (CS) emerged as a signal processing technique to reconstruct a signal sampled below the Nyquist sampling rate. We reconstruct a number of commonly used images using the compressed sensing method in this paper. We exploit the sparse nature of the images and reconstruct those using very few measurements. As a requirement, we have regarded the signal to be sparse either in its original domain or in some other domain. In this paper, we also experimented image reconstruction employing the compressed sensing techniques with the aid of either discrete wavelet transform (DWT) or the discrete cosine transform (DCT). We employ the basis pursuit algorithm and the Douglas-Rachford iterative algorithm for reconstructing the images. Basic theoretical and mathematical concepts underlying compressed sensing are applied. We further compare the reconstruction of the images using some well-known image quality metrics.**

Keywords— compressed sensing, acquisition method, reconstruction algorithm, convex optimization.

I. INTRODUCTION

Compressed sensing (CS) has emerged as a beneficial signal processing method [1–3], which allows us to reconstruct signals using fewer random measurements. We know from the famous Shannon-Nyquist sampling theory [4] that, for perfect reconstruction of a signal from samples, the sampling rate of a signal has to be equal to or greater than twice the maximum bandwidth of the original signal. In many image and video applications, this results in a huge number of samples, and we become obliged to compress the data. In other applications, for example, the radars and analog-to-digital converters, this huge sampling rate is not at all cost-effective [5], [6]. The discrete cosine transform (DCT) or the discrete wavelet transform (DWT) are usually employed for image and video coding and compression [7–9]. We use these methods for converting a high-definition image to a suitable basis where the image is sparse in nature. We then throw out most of the coefficients and only keep the most important coefficients. Then these coefficients can be inverse transformed to reproduce the high-quality image. The reconstruction based on Nyquist sampling is limited by the bandwidth of the image or video signal. By contrast, CS is based on the principle of reducing the number of measurements required [10], [11].

To specifically understand the process of CS, we can see the classic image compression mechanism as converting a high-resolution image into smaller numbers of data [12–14]. As such, we have to consider a large amount of data. For the sake of avoiding this large data set, compressive sensing directly measures the important data points, thereby substantially reduces initial measurements [3], [8], [15]. In this technique, we take the massive down-sampled image to begin with. Then we consider the non-zero coefficients of the suitable basis and inverse-transform these coefficients back to the high-quality image. Under the condition that the signal is sparse, it may be reconstructed by far lesser samples. A signal vector is termed as K-sparse, if it has a K-number of nonzero elements. This process can be divided into two sections acquisition and reconstruction.

CS is applied in numerous science engineering and medical fields It has the advantages of requiring fewer sensors and less memory while facilitating high-rate data transmissions. It has got applications in medical imaging, for example, in computed tomography (CT) imaging and/or in magnetic resonance imaging (MRI) [16]. This imaging using CS has the advantages of less acquisition time, less radiation exposure and a large number of patients can be supported at the same time. There are some advanced researches as well for improved imaging via CS. For example, the block sparsity-based image recovery technique may be applied.

Against this backdrop, in this paper, we investigate some CS based image reconstruction technique for the reconstruction of some popular images. We compare some characteristics of the reconstructed images, namely the peak signal-to-noise ratio (PSNR), the mean-squared error (MSE), and the structural similarity index measure (SSIM). We also compare the reconstructed image visually.

We organize the rest of the paper as follows. Section II provides an outline of the CS method and the relevant measurement matrix construction and the reconstruction strategy. Section III provides the results and discussion. Finally, we conclude in Section IV.

II. CS-BASED IMAGING

Here, an overview of CS based image reconstruction technique is provided.

A. *CS Technique*

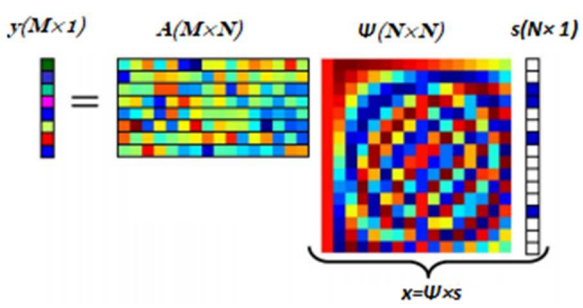


Fig. 1. Matrix Representation of CS, as appeared in [2], [15]

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The CS-based image reconstruction may be described in terms of Fig. 1. Fig. 1 is basically the matrix-based representation of the CS technique, where y denotes the measurement vector, Ψ indicates the transformation basis [2] s represents the sparse vector of coefficients., whereas A refers to the sensing matrix. To be specific, let us consider an N-pixel image to be represented by a $N \times 1$ vector x. We take M random samples (M<<N) and represent it by [15]

$$y = Ax \tag{1}$$

The image vector x can also be expressed in a sparse domain as [15]:

$$\mathrm{x} = \Psi \mathrm{s}, \tag{2}$$

where, s is the $N \times 1$ vector and is sparse in nature, Ψ is termed as the basis function, and we have:

$$\mathrm{y} = \mathrm{A}\Psi\mathrm{s} \qquad [\mathrm{x} = \Psi\mathrm{s}] \tag{3}$$

and

$$y = \phi\mathrm{s} \qquad [\phi = \mathrm{A}\Psi] \tag{4}$$

The CS based approach dictates us to solve for:

$$s = \phi^{-1} * y\,. \tag{5}$$

Since (5) denotes an underdetermined system of equations, there are essentially an infinite number of solutions for the sparse vector s.

This problem may appear impossible. However, with prior knowledge of s being sparse, it can be solved using l_1 norm-based optimization [5]. We can minimize the l_2 norm or the l_0 norm [5]. However, the l_2 norm minimization or least mean square does not exploit the sparsity of the signal. On the other hand, the l_0 norm minimization procedure is NP-complete and is not feasible in real time. Hence the l_1 norm-based optimization stands out to be the best choice.

The l_1 norm-based optimization [5] procedure is that we have to solve for s which has the minimum l_1 norm that satisfies the condition: $s = \phi^{-1} * y$. The optimization problem may thus be formulated as [5,15]:

$$\hat{s} = \arg min \|s'\|_1 \tag{6}$$

subject to the constraint:

$$\phi\mathrm{s}' = \mathrm{y}. \tag{7}$$

B. *Transformation Basis*

For the sake of attaining signal sparsity, the image/signal is transformed to an appropriate transformation basis. Natural images have sparse representation in DCT domain to some extent and have better sparsity in wavelet domain. For voice signal, the signal is sparse in Fourier transform domain, and can be expressed by a few number of Fourier coefficients. Radon transform is the most suitable transformation basis for medical images [6].

C. *Measurement Matrix*

A proper measurement matrix is necessary for perfect reconstruction. There are two properties to be satisfied for the matrices A and φ. The first requirement is that φ must satisfy a property called the restricted isometry property (RIP) [10]. The second one is that A must be incoherent' with respect to the sparsity basis function ψ. Random matrices are by nature incoherent. However, the problem with random matrices is that they cannot be reproduced at the receiver. So, it has to be transmitted with the signal for the appropriate reconstruction of the signal at the receiver. To circumvent the problem, Romberg proposed the random convolution approach. The first row of the matrix is generated at random. The second one is generated by a circular shift of the first one. The process repeats for the rest of the rows and generate random measurement matrices. The measurement vector y is generated by the convolution of the measurement matrix A with the original signal x. The measurement matrix can provide faster acquisition, easier storage and communication.

D. *Reconstruction using CS*

CS-based reconstruction algorithms find an estimated sparse solution of the original signal/image in a sparse domain. Many researchers are working to lessen the number of measurements, complexity, and noise, thereby increasing the reconstruction speed, and performance. There are various types of algorithms developed for CS reconstruction. One of them is the convex optimization approach [2], [13], [15].

1) Basis Pursuit (BP): The basis pursuit method [11] finds the minimum l_1 solution that solves the equation (4) mentioned above. As $y = \phi s$ is an underdetermined system of equations, there are infinite number of s that satisfy the equation. If we assume that, in the basis domain, the solution is sparse, then we find a l_0 norm, which gives the sparsest solution. However, as already mentioned, the nature of the problem makes the solution computationally non-feasible as the model becomes larger. By contrast, the l_1 optimization problem is convex by nature and it tends to give a sparse solution. The optimization procedure can be expressed as:

$$s = arg\ min\big||s|\big|_1, \quad \text{subject to } \mathrm{y} = \phi\mathrm{s} \tag{8}$$

2) BP for noisy signal: This algorithm finds the minimum l_1 solution, however, takes the noise into consideration. This method does not impose the optimization constraint because it takes account of the noisy signal. According to this algorithm, in order to get the minimum l_1 solution, the l_2 norm of the squared error between y and ϕs has to be equal or smaller than a constant Ɛ, whereas the term λ makes a flexible balance between the error and the sparsity [11]. This optimization problem may be stated as:

$$s = \arg min\, \lambda\big||s|\big|_1, \text{ subject to, } \tfrac{1}{2}||\mathrm{y} - \phi\mathrm{s}||_2^2 \le \varepsilon \tag{9}$$

3). CS Reconstruction Using Operator Splitting: The optimization problem mentioned in (9) of Section II-C.2 can be reduced to its equivalent form given by:

$$min\, \tfrac{1}{2}||\mathrm{y} - \phi\mathrm{s}||_2^2 + \lambda\big||s|\big|_1, \tag{10}$$

which may again be expressed by [12]:

$$min\, F(x) + G(x)$$

subject to the constraints given by

$$F(x) = \tfrac{1}{2}\big||\mathrm{y} - \phi\mathrm{s}|\big|_2^2, \text{ and } \quad G(x) = \lambda||s||_1. \tag{11}$$

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The above may be solved by using the Douglas-Rachford method [12]:

$$\tilde{x}_{k+1} = \left(1 - \frac{\mu}{2}\right)\tilde{x}_k + \frac{\mu}{2}\mathrm{rPox}_{\gamma G}\left(\mathrm{rProx}_{\gamma F}(\tilde{x}_k)\right), \quad (12)$$

and $$x_{k+1} = \mathrm{Prox}_{\gamma F}(\tilde{x}_{k+1}). \quad (13)$$

III. RESULTS AND DISCUSSION

We have investigated the CS based image reconstruction by employing the wavelet basis function, the DCT based basis function and by using the Douglas-Rachford scheme. We have also compared the CS based image reconstruction methods in this section.

A. CS in wavelet domain

We have studied the CS reconstruction of the "flower" image using a MATLAB 2021a simulation platform. For computational purpose the image is converted to a 50 x 50 pixel image. Two-level wavelet transformation [7] is done using Daubechies 2 ("db2") mother wavelets [8]. The approximate image is 14 x 14 pixels which contains low-frequency component of the image. The rest of the detailed sub-signals are sparse. More sparsity is enforced by forcing the smaller number of coefficients to become zeros. The sparse signal which contains the detail sub-signal is convoluted by a structured random measurement matrix, thereby giving us the (M x 1)-sized vector y. Then the y vector along with the approximate image are transmitted. This is solved by the b pursuit method discussed above for reconstructing the detailed sub-signal. The approximated image and detailed sub-signal then help in reconstructing the image using the inverse discrete wavelet transform (IDWT). The images reconstructed from approximate image (196 pixels) and detailed sub-signal which is solved for a distinct number of measurements are shown in Fig. 2. As a fair comparison of the BP and the least squares (LS) methods, Table I illustrates the number of measurements together with the image quality metrics achievable by both methods.

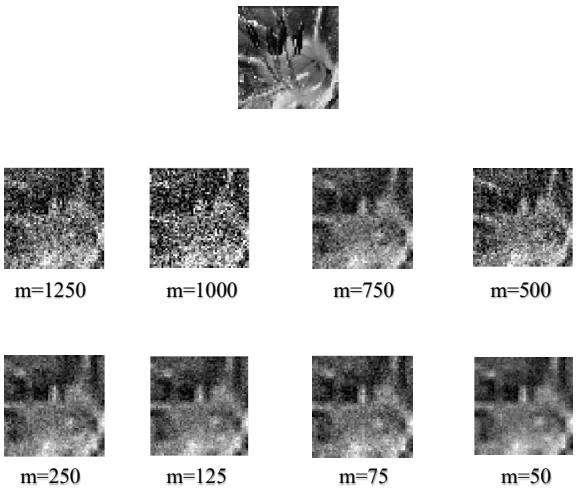


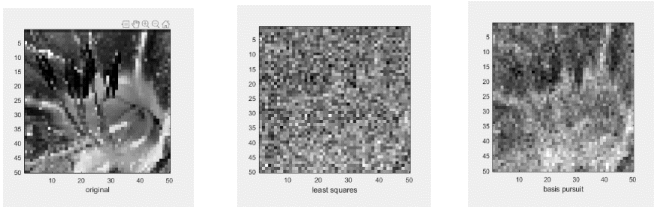
Fig. 2. Reconstructed images for different values of measurement.

Table I: MSE, PSNR, and SSIM values for reconstruction of flower image in wavelet domain from different measurements using BP. & LS.

Algorithm	*Meas.*	*MSE*	*PSNR (dB)*	*SSIM*
B.P.	1000	0.0529	16.198354	0.9991572
	500	0.0374	15.994066	0.9991719
	125	0.0334	17.419336	0.9994867
L.S.	1000	2.8157	9.0978834	0.9900335
	500	4.8656	7.9094318	0.9832735
	125	6.8448	7.1541157	0.9773526

B. CS in DCT Domain

The same "flower'' image is again transformed to DCT-domain [9], and then convoluted with a random measurement martix and generates a much smaller vector. This vector is then transmitted and at the receiver side it is solved by basis pursuit algorithm. The image in Fig. 3 shows recovery performance for least square and basis pursuit algorithm for 1250 measurements. We also notice from Fig. 3 that basis pursuit provides better quality reconstructed image than least squares. However, we know that natural images are not that much sparse in DCT domain and hence CS based reconstruction is not as efficient as that in the wavelet domain. We compare the image quality metrics of the reconstructed flower image by the BP and the LS technique in Table II using DCT-based reconstruction.



Original Least Squares Basis Pursuit

Fig. 3. Least Square V/S Basis Pursuit recovery

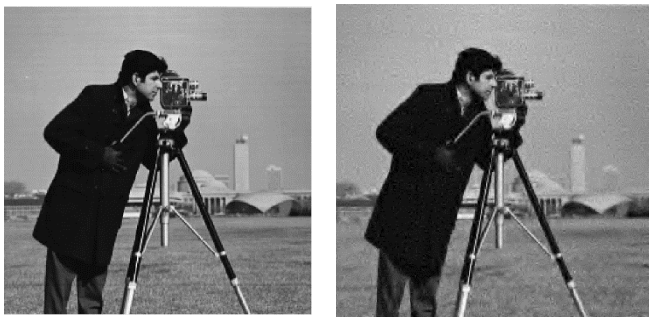
Table II: MSE, PSNR, and SSIM values for reconstruction of flower image in DCT domain from different measurements using BP and LS.

Algorithm	*Meas.*	*MSE*	*PSNR (dB)*	*SSIM*
BP.	1250	0.0217	18.469850	0.9996154
	750	0.0530	15.844793	0.9991441
	250	0.1408	13.331563	0.9978923
LS.	1250	1.9349	10.840431	0.9929038
	750	3.8509	8.4278370	0.9866532
	250	6.3021	7.3181430	0.9790112

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C. Reconstruction using Douglas-Rachford scheme

For illustrating the CS-based image reconstruction by the Douglas-Rachford (D-R) scheme [12], we use the "cameraman'' image having 65,536 (256 x 256) pixels. The image is first transformed to wavelet domain, and the high-frequency sparse detailed sub-signal is convoluted by a structured random measurement matrix. The minimum l_1 recovery is performed on the combined signal, and the recovered signal is then solved by the D-R iterative scheme. The image is reconstructed from 4,096 linear and 16,384 CS measurements using the D-R scheme as shown in Fig. 4.



Original Image Reconstructed Image

Fig. 4. Reconstruction using Douglas-Rachford scheme.

We have also reconstructed the different popular images using the D-R scheme. We have computed the image quality metrics of the images reconstructed by the D-R scheme, and tabulated the values of these parameters in Table III. We observe from Table I that the DR scheme can be effectively employed for reconstructing images and the reconstruction technique is efficient in providing better quality images.

Table III: MSE, PSNR, and SSIM values for CS reconstruction of some standard images using D-R iteration method

Name	*MSE*	*PSNR (dB)*	*SSIM*
Lena	0.0043	27.8358454	0.9999643
Man	0.0050	24.8408299	0.9999260
Pepper	0.0107	36.0359747	0.9999983
Barbara	0.0138	25.1859807	0.9999122
Boat	0.0033	32.9927270	0.9999966
Baboon	0.0115	19.8011264	0.9997534
Rubik	0.0007	42.0974646	0.9999996
Monalisa	0.0005	40.9260703	0.9999995

IV. CONCLUSION

We have effectively employed CS for reconstructing images in this paper. CS depends on the sparsity of images in some domains, and sparsity depends on the choice of the dictionary. By observing the results, it can be inferred that discrete wavelet transform performed better than discrete cosine transforms as the basis function because it is the wavelet domain where natural images are sparser. Among the l_0 , l_1 , and l_2 norms, l_1 norm is the most efficient. Furthermore, the D-R iterative scheme provided improved image reconstruction compared to the BP algorithm. This is a very rudimentary study on the reconstruction of images by the CS method. There are some advanced image processing methods, with which we have not compared our results. However, the development of improved CS techniques for reconstructing images, especially medical images, is included in our potential future study.

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