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Grover's Quantum Search for the Signal Detection of OFDMA Downlink and SC-FDMA Uplink

Swaraj Mondal
ECE Discipline
Khulna University
Khulna-9208, Bangladesh
swarajmondal569@gmail.com

Nazmun Nahar Zarin
ECE Discipline
Khulna University
Khulna-9208, Bangladesh
nazmun.nahar1998@gmail.com

Mohammad Ismat Kadir
ECE Discipline
Khulna University
Khulna-9208, Bangladesh
ismat.kadir@ece.ku.ac.bd

***Abstract*—There are different access systems proposed for next-generation wireless communications. As the number of users in wireless systems increases, the challenge of reliable communications can be faced with the aid of exhaustive search-based signal detection. However, exhaustive maximum-likelihood searches impose a high complexity. In classical communication systems, performance improvement is achieved at the cost of substantially increased computational complexity. In this study, we consider an OFDMA downlink and SC-FDMA uplink scenario. We propose the employment of quantum search methods for signal detection. Specifically, we employ Grover's algorithm for database search in signal detection. We observe from our simulations that the error performance provided by the classical ML search is attainable by our method. As a benefit of using quantum search, the complexity imposed is substantially reduced compared to using classical ML detectors. We also compare the performance of the proposed scheme with Grover's search-based MC-CDMA detector. We investigate with multiple users as well and find that the quantum search-based scheme is effective in improving performance, while the computational burden is very low.**

***Index Terms*—Grover's algorithm, SC-FDMA, OFDMA, maximum likelihood estimation, computational complexity.**

I. Introduction

Wireless communications have been constantly adopting new access technologies to meet the increasing user demands. With the increasing number of users, the need for reliable, secure, and low-complexity access methods is also increasing. To provide spectrum allocation to these increasing numbers of users, an appropriate multiple-access scheme is essential. Orthogonal multiple access techniques were considered up to the fourth-generation (4G) wireless systems. With the advent of 5G and beyond wireless systems, non-orthogonal multiple access (NOMA) schemes are attracting increasing interest. In the code domain, multicarrier-based multiple access methods are multi-carrier code division multiple access (MC-CDMA) [1] and its variants. A multiuser version of orthogonal frequency division multiplexing (OFDM) is the orthogonal frequency division multiple access (OFDMA), where each user is allocated a portion of the available subcarriers. Another multiple access technique conceived mainly for the uplink scenario is single-carrier frequency division multiple access (SC-FDMA) [2], where an additional discrete Fourier transform (DFT)-based precoding is employed in the transmitter. MC-CDMA allows multiple users to share a common frequency band using multiple subcarriers and an appropriate spreading sequence [1]. However, as with any CDMA system, MC-CDMA is also interference-limited and may suffer from severe multi-user interference (MUI) [2]. Apart from the MC-CDMA-like spread spectrum schemes, OFDMA, and SC-FDMA are also prominent multiple access mechanisms. Specifically, OFDMA was under consideration for the downlink, while SC-FDMA was considered for the uplink in the fourth-generation (4G) wireless networks. In particular, SC-FDMA offers enhanced performance in uplink frequency-selective environments. The transmitted signal of SC-FDMA is also notably reduced peak-to-average power ratio (PAPR) [2], [3]. Despite ongoing exploration into advanced multiple access methods, NOMA is studied to address the requirements of fifth-generation (5G) and subsequent wireless systems, although comprehensive research into its complete functionality is still in progress.

The signal detection of these access methods is important for proper communication. Different signal detection strategies are available in the literature. For example, linear zero-forcing (ZF) equalization-based detectors can be used. As a substitute, minimum mean-squared error (MMSE)-based detectors are also the basic candidates [3]. There are message-passing detectors and maximum likelihood (ML) detectors and these detectors can be either hard-decision or soft-decision-based. Among all these detectors, the ML detector has the optimal error performance, albeit the detection complexity it imposes is the highest. This complexity is severely increased if an ML multiuser detector (MUD) is used. Thus, a crucial trade-off arises to provide better error performance at a reduced detection complexity.

Motivated by the low complexity imposed by quantum search strategies, there exist few works on applying the low-complexity quantum search in place of ML search-based signal detection method [4], [5]. To be specific, the quantum Grover's search imposes an $O\left(\sqrt{N}\right)$ complexity compared to the $O\left(N\right)$ in classical search for searching over N components [5], [6]. There exist extensions to Grover's search algorithm as well [7]. Grover's algorithm can be used in the detection of almost all communication schemes. It has been

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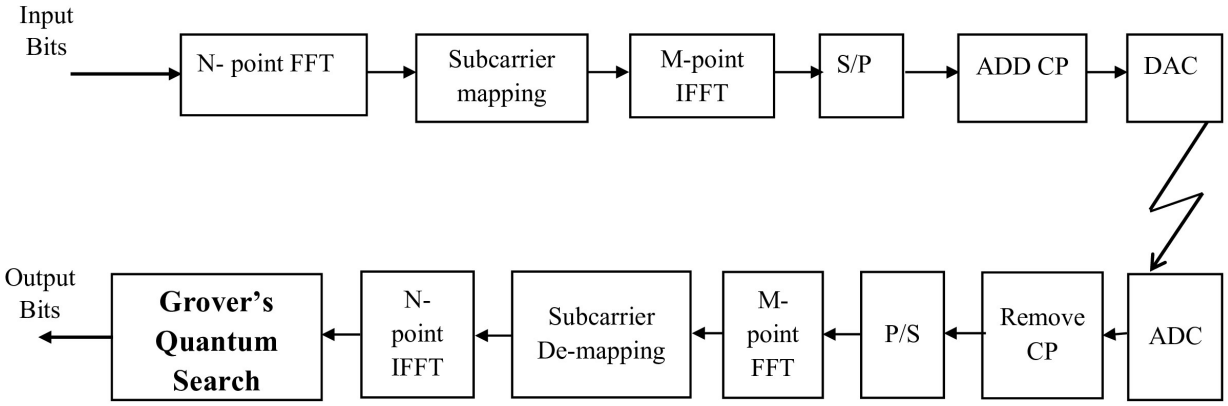


Fig. 1. Transceiver architecture of the Grover's algorithm-aided SC-FDMA scheme. The scheme will become an OFDMA system if there is no N-point DFT block and the same-length IDFT block in the transmitter and the receiver, respectively.

preliminarily used in the MC-CDMA signal detection in [8]. We extend the application of Grover's search in the downlink and uplink signal detection by using OFDMA and SC-FDMA, respectively.

The contributions of this article can be outlined as:

1) Grover's quantum search algorithm is applied to perform signal detection for downlink using OFDMA and in the uplink using SC-FDMA.
2) The performance versus complexity trade-off is investigated for the proposed scheme. The effect of MUI is also studied for the proposed scheme.

Now let us provide the organization of the remainder of this paper. An overview of the communication architecture is provided in Section II. The quantum search-based detection method is detailed in Section III. Numerical results are presented in Section IV. Section V discusses the limitations of this research. Finally, the paper is concluded in Section VI.

II. System Overview

A. The Transmission Scheme

The schematic diagram of the SC-FDMA transceiver is depicted in Fig. 1. If the initial N-point DFT operation block in the transmitter and the final N-point IDFT operation block in the receiver are omitted from the diagram, it represents an OFDMA system. At the transmitter side, the input bits of user u are first mapped to symbols denoted by $\boldsymbol{x}^u$, where there are a total of U users. Then these symbols undergo an N-point DFT operation. The frequency-domain (FD) symbol sequence $[X^u(1), X^u(2), \cdots, X^u(N)]^T$ represented as $\boldsymbol{X}^u$ is obtained by using the DFT. These FD symbols are assigned to the subcarriers. The subcarriers can be assigned either in a localized or in an interleaved manner. Following the subcarrier mapping, we execute the M-point inverse DFT (IDFT) operation. Subsequently, a cyclic prefix (CP) of suitable length is appended to alleviate intersymbol interference. The signal is then converted from digital to analog and finally transmitted over the wireless channel.

B. The Receiver

The analog signal received is transformed to digital form at the receiver. After removing the cyclic prefix, the data symbols undergo an M-point DFT operation. The frequency-domain output can be represented by

$$\boldsymbol{Y} = \sum_{u=1}^{U} \tilde{\boldsymbol{H}}^u \tilde{\boldsymbol{X}}^u + \boldsymbol{V} = \sum_{u=1}^{U} \tilde{\boldsymbol{H}}^u \boldsymbol{X}^u + \boldsymbol{V}, \tag{1}$$

where $\boldsymbol{V}$ denotes the frequency-domain additive white Gaussian noise (AWGN), while $\tilde{\boldsymbol{H}}^u$ represents the frequency response of the channel. After completing these receiver operations, the signal undergoes equalization using either a zero-forcing (ZF), or a minimum mean squared error (MMSE) linear equalizer. Alternatively, a decision feedback equalizer can also be employed. The subcarriers are then demapped and an N-point IDFT operation is performed on each user's signal. The signal $\boldsymbol{z}^u$ is then used to estimate the user's transmit signal. An ML search can be used for this estimation [2], [3]:

$$\hat{\boldsymbol{x}}^u = \underset{\boldsymbol{s^u}[n]\in\boldsymbol{S}}{\arg\min} \sum_{n=1}^{N} \left\| \boldsymbol{z}^u[n] - \sum_{u=1}^{U} \tilde{\boldsymbol{H}}^u \boldsymbol{s^u}[n] \right\|^2 \tag{2}$$

where $\boldsymbol{S}$ is the set of constellation symbols. If the proposed system represents an OFDMA system, then both the N-point DFT in the transmitter and the N-point IDFT in the receiver of Fig. 1 will be absent. As a result, we can use a linear FD equalizer followed by an ML search or a straightforward ML estimation given by

$$\hat{\boldsymbol{X}}^u = \underset{\boldsymbol{X}^u[n]\in\boldsymbol{S}}{\arg\min} \sum_{n=1}^{N} \left\| \boldsymbol{Y}[n] - \sum_{u=1}^{U} \tilde{\boldsymbol{H}}^u \boldsymbol{X}^u[n] \right\|^2. \tag{3}$$

The ML search expressed by (2) or (3) provides an optimal signal estimation. However, the computational complexity associated with the above-mentioned ML search is high. Hence, we opt for a quantum-inspired Grover quantum search for detection and enhance the efficiency of the search and speed of the search process with less complexity without sacrificing

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the performance of the optimal ML detector.

The theoretical BER of the above-mentioned scheme can be expressed [2], [3] for AWGN channel by

$$P_b = \frac{1}{2} erfc\left(\sqrt{\gamma}\right) \tag{4}$$

and, for the Rayleigh fading channel by

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma}{1+\gamma}}\right), \tag{5}$$

where $\gamma = \frac{E_b}{N_0}$ represents the channel signal-to-noise ratio (SNR).

The process of searching data on Grover's search-based detector follows the basic steps of Grover's algorithm. Grover's search is applied here to estimate the symbols transmitted. As previously noted, the complexity introduced is notably less than that of the ML detector.

III. Detection Using Grover's algorithm

We note that the detection rule dictated by (2) is an exhaustive ML search. To employ a reduced-complexity detector, we employ the quantum Grover's search. Specifically, we store the values of the Frobenius norm given by (2) and then find the minimum using the algorithm of [5], [6].

At this point, we make it clear that previous studies on applying Grover's algorithm for wireless signal detection do not mention how they employ the quantum algorithm for finding the index of the minimum value in the database of the Frobenius norm mentioned above. Grover's algorithm is the quantum algorithm for searching a specific item in a database. Our objective is to find the minimum item. Hence, the detection algorithm comprises two parts - finding the minimum and searching the database for its index. Thus, our proposed method employs the method stated in [5] and the algorithm of [5] uses Grover's algorithm of [6].

The procedure is thus illustrated in terms of two algorithms: Algorithm 1: Proposed Grover Search-based Detector, and Algorithm 2: Grover's Search-based Algorithm.

Algorithm 1: Proposed Grover's Search-based Detector.

Input: Received symbol stream, Set of constellation symbols $\boldsymbol{S}$
Final Estimated symbol stream

1) Let the search space be M. Choose a threshold index ℓ, such that $1 \leq \ell \leq M$.
2) Do until number of iterations is greater than $22.5\sqrt{M} + 1.4\log_2 M$. Then jump to (c).
   a) Memory initialization: $\sum_k \frac{1}{\sqrt{M}}|k\rangle|\ell\rangle$.
   Take all terms k, for which $D(k) < D(\ell)$, where $D(.)$ represents the database entry.
   b) Apply the Grover's search Algorithm 2:.
   c) Take the first outcome ℓ'.
   if $D(\ell') < D(\ell)$, set ℓ' to ℓ.
4) Take the index ℓ.
5) Estimate the symbols corresponding to the solution indices.

Thus we see that the Algorithm 2: Grover's Search-based Algorithm is called from within Algorithm 1: Proposed Grover Search-based Detector.

Algorithm 2: Grover's Search-Algorithm.

1) Begin with a register containing n qubits, n qubits. Initialize each to state $|0\rangle$;
2) Employ the Hadamard transform $\boldsymbol{\mathcal{H}}$ to every state within the register to prepare the superposition states using (6);
3) for $i = 1$ to $\frac{\pi}{4}\sqrt{N}$, do the following:
   a) Apply the oracle operator U_w on the register states;
   b) Apply the Hadamard transform $\boldsymbol{\mathcal{H}}$ to every state within the register;
   c) Apply the inversion about mean operator U_s of (10);
   d) Apply the Hadamard transform $\boldsymbol{\mathcal{H}}$ to every state within the register;
4) Measure the register to find a solution with high probability;

This procedure of Algorithm 2: Grover Search-Algorithm can be summarized as follows.

The set of values obtained mentioned above is the database over which the quantum search is applied. We consider a quantum register that stores qubits into their superposition states using the Hadamard gate.

We start with a register of $n = \log_2 N$ qubits and then apply a quantum superposition using the Hadamard gate. The superposition state by applying the Hadamard transform on the state $|\Psi_0\rangle = |0\rangle^{\otimes n}$ can be expressed [4]–[6] as

$$|\Psi_1\rangle = \mathcal{H}^{\otimes n}|0\rangle^{\otimes n} = \frac{1}{\sqrt{N}} \sum_{j=0}^{(N-1)} |j\rangle \tag{6}$$

where $N = 2^n$ represents the total number of potential states, with $0 \leq j \leq (N-1)$. We use an oracle operator and a diffusion operator [5], [6]. We introduce an oracle that marks the solution by inverting the sign of the amplitude of the target state and the oracle can be represented [4]–[6] as

$$U_w|x\rangle = (-1)^{f(x)}|x\rangle \tag{7}$$

where $|w\rangle$ represents the unknown target state and

$$|x\rangle = \begin{cases} 1 & \text{if } x \text{ is a solution} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

The oracle essentially identifies the solution without providing any specific information about it Then we apply a series of operations to amplify the amplitude of the mark state and suppress the amplitude of the other states. We utilize the Grover operator G to boost the amplitude of the target states.

$$G = U_s U_w, \tag{9}$$

where U_s is the inversion about the mean operator and U_w is the oracle from above. The inversion about the mean operator is expressed [4]–[6] by

$$U_s = 2|\Psi_1\rangle|\Psi_1\rangle - I, \tag{10}$$

where $|\Psi_1\rangle$ is the superposition states obtained above.

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Table I
MAIN SIMULATION PARAMETERS

Parameter	Value
Equalization	ZF, MMSE, ML
M	64
N	16
Modulation order	2, 4
Used channel model	Frequency-selective channel
Fading envelope	Rayleigh distributed
No. of channel taps	4

Now we apply U_s to the quantum state

$$|\Psi_1\rangle \xrightarrow{U_s} U_s|\Psi_1\rangle. \tag{11}$$

This operation reflects the amplitude about the mean amplitude. We repeat these steps approximately $\sqrt{N}$ times. The optimal number of iterations [4]–[6] is roughly $\frac{\Pi}{4}\sqrt{N}$. After the desired number of iterations, measure the quantum state $|\Psi_1\rangle$, which provides the estimated signal.

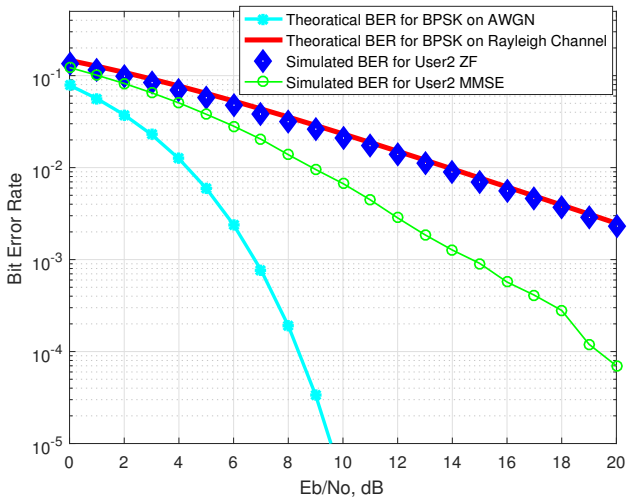


Fig. 2. Error performance of SC-FDMA system for linear equalization-based detection for a single-user scenario.

IV. NUMERICAL RESULTS

The error performance of the SC-FDMA and OFDMA system using Grover's quantum search-based detector is investigated here. This is also compared with that of ZF, MMSE, and ML detectors. We use $N = 16$ and $M = 64$ for our simulations and we consider a 4-tap Rayleigh fading channel which is frequency-selective. We also investigate the system's performance for different numbers of users to study the effect of MUIs. The primary simulation parameters are outlined in Table I

We investigate the BER characteristics of the proposed SC-FDMA system in Fig. 2. The BER value at lower SNR is almost the same for ZF-equalized and MMSE-equalized

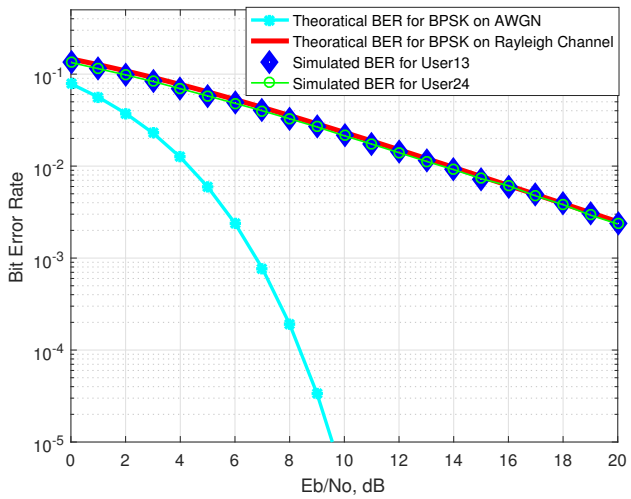


Fig. 3. Error performance of the SC-FDMA system with two different users.

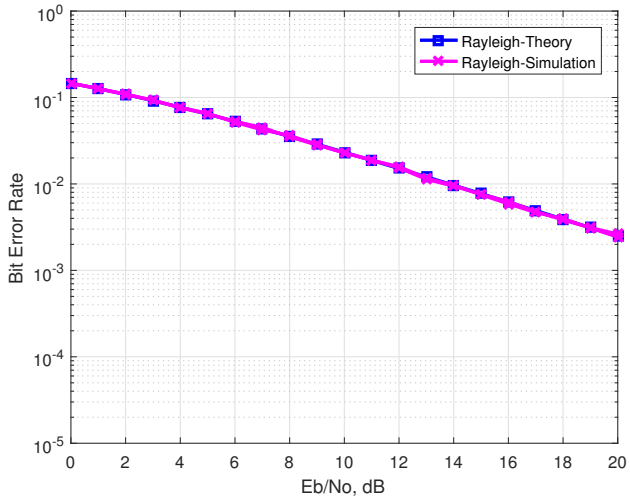


Fig. 4. BER performance of Grover's search-assisted OFDMA system.

detectors. As expected, the error performance is improved, i.e. the BER decreases with the increase in SNR. However, as predicted, the rate is not equal for ZF and MMSE. At 20 dB SNR, the BERs for ZF and MMSE are 0.002 and 0.0001, respectively. That means that MMSE exhibits better BER performance than ZF.

Fig. 3 represents the BER analysis of the proposed system for two different users of the SC-FDMA system, which shows identical performance for each of the users. For number of bits 10^5, the initial BER at SNR 0 dB for user13 and user24 is 2×10^{-1} and at 20 dB SNR value. To compare with the OFDMA system, Fig. 4 shows the error performance of Grover's search-based OFDMA system.

Fig. 5 shows the error results of Grover's search-aided OFDMA system for two and four users. It shows the BER performances to be identical, indicating that MUI in OFDMA

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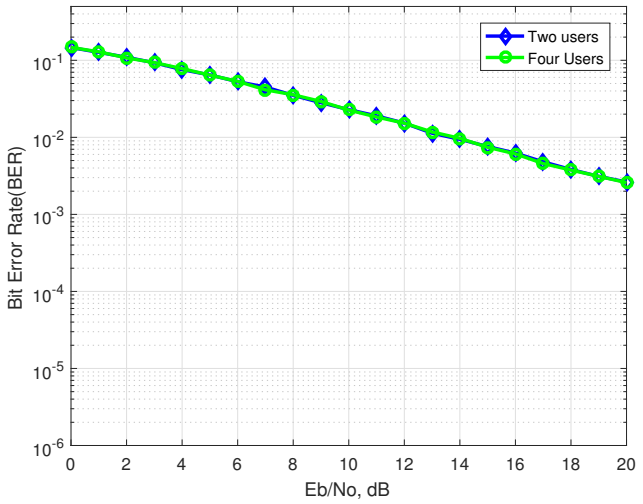


Fig. 5. Error performance of Grover's search-based OFDMA system when the total number users are 2 and 4.

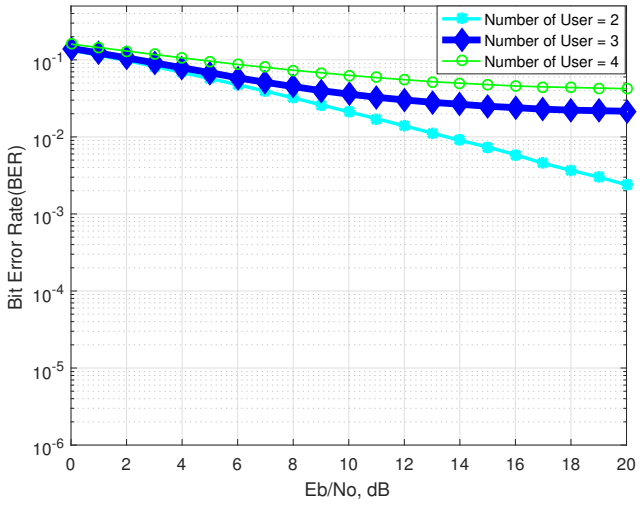


Fig. 6. Error performance of Grover's search-based MC-CDMA system with different total number of users in the system.

is almost insignificant. For a comprehensive comparison, Fig. 6 depicts the error performance of the quantum-assisted MC-CDMA system under different numbers of users. As depicted, the bit error rate (BER) of the multiple access system deteriorates with an increase in the number of users. Initially, at an SNR value of 0 dB, the BER remains relatively consistent regardless of the number of users. However, as the SNR value increases, the BER decreases more gradually when the number of users is higher.

Fig. 7 compares the BER attainable using the ML-search based detector and Grover's search-aided detector. As indicated in Fig. 7, there is essentially no performance penalty in using the quantum Grover's search for the detection. Grover's search-based detector exhibits optimal performance similar to

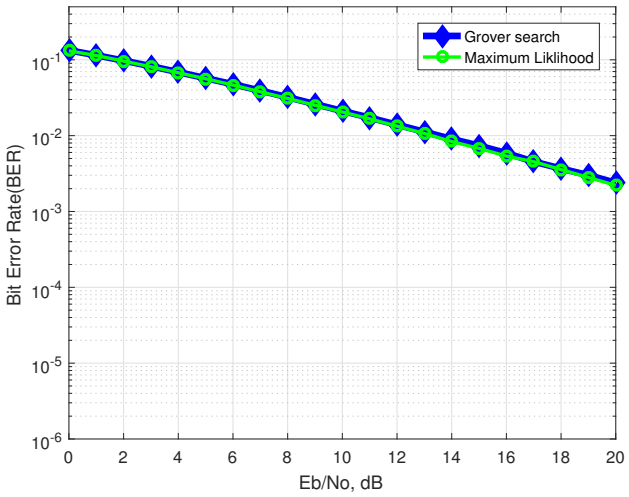


Fig. 7. BER comparison of ML- and Grover's search-based SC-FDMA detectors.

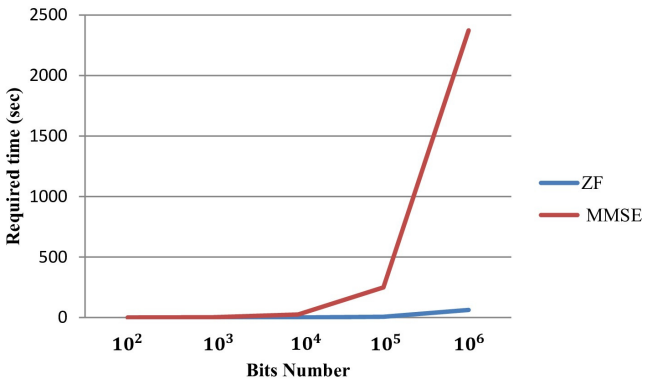


Fig. 8. Complexity Comparison of Zero Forcing and MMSE equalization.

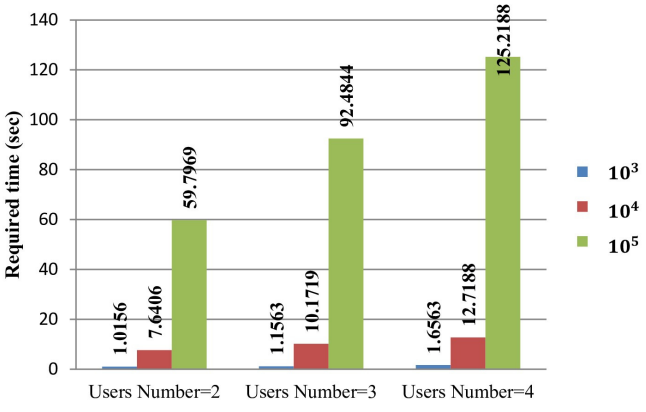
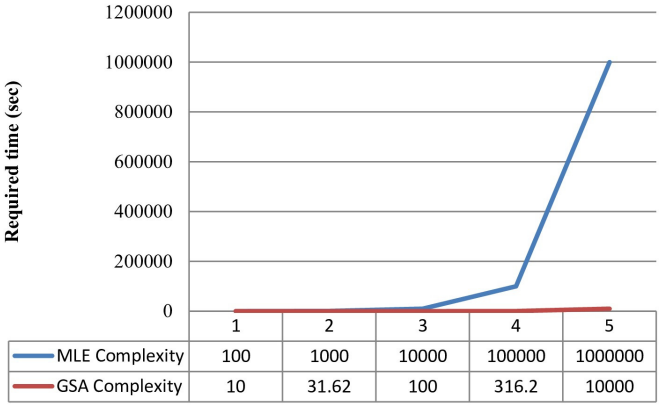


Fig. 9. Complexity Comparison when the number of users as well as number of source bits is different.

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	1	2	3	4	5
MLE Complexity	100	1000	10000	100000	1000000
GSA Complexity	10	31.62	100	316.2	10000

Fig. 10. Computational complexity of the relevant detectors.

the ML detectors, although at a reduced complexity.

Fig. 8 portrays the complexity of the scheme measured according to the simulation runtime (CPU time) for ZF and MMSE equalization-based detectors. We see that the simulation time for the MMSE-based detector is higher than that of the ZF-based detector. The ZF-based is seen to impose the least complexity, as seen from Fig. 8.

The impact of varying the number of users and increasing the transmitted source bits is illustrated in Fig. 9. It is evident from the figure that a higher number of users necessitates more CPU time. As expected, with a larger number of source bits, the required simulation time also increases.

Fig. 10 provides the comparative complexity imposed by using the proposed quantum search-aided scheme versus the ML detector. As depicted in Fig. 10, the complexity of the ML-detector escalates with both the number of users and the quantity of source information bits. By contrast, Grover's search-based detector provides a reasonable complexity under different scenarios.

V. Limitations of This Study

This is a preliminary study on applying Grover's search as a substitute for ML search for OFDMA and SC-FDMA signal detection. There are more advanced quantum searches than Grover's search for applications in the detection of wireless signals. There are also advanced multiple access methods. These multiple access methods sometimes require ML multiuser detectors (MUDs). The ML MUDs impose an extremely high computational complexity. Advanced Grover's search or another advanced quantum search method can solve the problem of high complexity in these applications.

VI. Conclusions

In this study, Grover's search-assisted detector has been proposed for detecting OFDMA and SC-FDMA signals which are employed in the downlink and uplink scenarios, respectively. The classical ZF-, MMSE- and ML-based detectors can be employed for the system considered. We see that ZF has less complexity but poor BER performance. MMSE can provide moderate performance - however, its complexity is somewhat higher compared to ZF. Conversely, the ML detector can deliver optimal performance - however, it comes with high detection complexity. Under these circumstances, the proposed quantum-inspired detector offers an optimal error performance with significantly reduced computational complexity. The proposed scheme can work under multi-user scenarios, and the MUI imposed is not severe. Thus the proposed scheme appears a promising candidate for signal detection. This study provides an idea of applying Grover's search in the signal detection of wireless multiple access schemes. The OFDMA and SC-FDMA were 4G multiple access candidates. The proposed quantum method can be extended to the emerging wireless access techniques of next-generation system. More advanced quantum search methods may also be employed in the future.

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